

Practice for Final Schneider
Always give reasons

1. (15 points) Write down the number of possibilities in the following problems (you can leave in symbolic form with numbers in correct places)
 - a. The number of different letter arrangements from **tessellate** – **This is a theorem we have**
 - b. How many different 7 card poker hands are there (you get dealt 7 cards to look at)---**52 choose 7**
 - c. How many different binary numbers are there of length less than or equal to 10. **$2^{11}-1$**
 - d. You have 16 people in a class. How many ways can they line up for lunch? **$16!$**
 - e. How many outcomes of 20 flips of a coin have less than 5 heads?---**20 choose 0 + 20 choose 1 + 20 choose 2 + 20 choose 3 + 20 choose 4**
2. (20 points) Fifty five percent of the students at a certain school wear neither a ring nor a necklace. Twenty percent wear a ring and forty percent wear a necklace. If one of the students is chosen randomly what is the probability that the student is wearing. **You are given that $P((R \cup N)^c) = .55; P(R) = .20; P(N) = .4$**
 - a. a ring or a necklace? **$P(R \cup N) = 1 - P((R \cup N)^c) = .45$**
 - b. a ring and a necklace?-- **$P(R \cap N) = P(R) + P(N) - P(R \cup N) = .2 + .4 - .45 = .15$**
3. (15 points) Suppose for the two events A, B we know $P(A|B) = .2$; $P(B) = .5$; and $P(AB^c) = .2$. Find $P(B|A)$. **$P(B|A) = P(BA)/P(A) = P(A|B)*P(B)/(P(A|B)*P(B) + P(AB^c))$.. plug in numbers**
4. (30 points) Suppose that a fair die is independently rolled twice.
 - a. What are the probability mass functions for the random variables R_1, R_2 that are the values of the first and second roll respectively? (Graph them). **Both pmf are the same and are height 1/6 for each integer 1,2,..., 6. Graph are 6 lines of height 1/6 at the pts 1,2,..6**
 - b. What is the cumulative density function of R_1 ? Graph it. **The cumulative function jumps by a 1/6 at pts 1,2, .. 6 and 0 to begin and 1 when ≥ 6 .**
 - c. Graph the probability mass function of the random variable $3*R_1$. **Height 1/6 at points 3,6,9,..18**
 - d. Graph the probability mass function of the random variable R_1-3 . **Height 1/6 at pts -2,-1,... 3**
 - e. What is $E(R_1)$ and $E(R_2)$? **We did this in class and they are each 3.5**
 - f. What is $Var(R_1)$ and $Var(R_2)$? **Again from class 2.916**
 - g. Consider the random variable $S = R_1 + R_2$ (the sum of the rolls) What is $P(\{S=6\})$? **5/36 as done in class**

- h. Consider the random variable $M=R_1 \cdot R_2$ (the product of the rolls). What is $P(\{M=6\})$? **1/9**
 - i. What is $E(S)$? **7**
 - j. What is $E(M)$? **$3.5 \cdot 3.5$ from theorem**
 - k. What is $\text{Var}(S)$? **$2 \cdot 2.916$ from theorem**
5. (30 points) Use theorems or definitions to show:
- a. $\text{Var}(aX) = a^2(\text{Var}(X))$ where a is some number. (do not use this fact to prove itself)
 - b. Give an example where $\text{Var}(X_1+X_2) \neq \text{Var}(X_1)+\text{Var}(X_2)$
 - c. Let X_1, X_2, \dots, X_n be independent random variables having a Bernoulli distribution ($P(\{X_i=1\})=p$; $P(\{X_i=0\})=q$):
 - i. what is the $E(X_i)$?
 - ii. what is the $\text{Var}(X_i)$?
 - iii. If $A = (X_1+X_2+\dots+X_n)/n$ what is the $E(A)$? (A is the average)
 - iv. What is the $\text{Var}(A)$?

These are just theorems from class, For C

- i) p**
- ii) pq**
- iii) p**
- iv) pq/n**